



General Certificate of Secondary Education
2024

Centre Number

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Candidate Number

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Further Mathematics

Unit 1 (With calculator)

Pure Mathematics



MV24

[GFM11]

TUESDAY 21 MAY, AFTERNOON

Time

2 hours, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.

You must answer the questions in the spaces provided.

Complete in black ink only.

All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.

Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.

Answer **all thirteen** questions.

Information for Candidates

The total mark for this paper is 100.

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

You may use a calculator.

The Formula Sheet is on pages 3 and 4.

Formula Sheet

Pure Mathematics

Quadratic
equations:

$$\text{If } ax^2 + bx + c = 0 \quad (a \neq 0)$$

$$\text{then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Differentiation: If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$

Integration: $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c \quad (n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log a^n = n \log a$$

Matrices:

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{then } \det \mathbf{A} = ad - bc$$

$$\text{and } \mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(ad - bc \neq 0)$$

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(Questions start overleaf)

1 Given that $\frac{dy}{dx} = 6x^2 - \frac{2}{5x^2} + 4$

(i) find an expression for $\frac{d^2y}{dx^2}$, [2 marks]

Answer _____

(ii) find an expression for y , given that
 $y = 4$ when $x = -2$ [5 marks]

Answer _____

2 Solve the equation $x^2 + 14x + 10 = 7$ by **completing the square**. [4 marks]

Give your answer in surd form.

Answer _____

3 Solve the inequality [4 marks]

$$(x + 1)^2 \geq 2x + 5$$

You **must** show clearly each stage of your solution.

Answer _____

4 (i) Solve the equation [2 marks]

$$\cos x = -0.6$$

for $-180^\circ \leq x \leq 180^\circ$

Answer _____

(ii) Hence solve the equation [3 marks]

$$\cos\left(\frac{\theta}{2} - 20^\circ\right) = -0.6$$

$$\text{for } -360^\circ \leq \theta \leq 360^\circ$$

Answer _____

5 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

(i) Find the matrix \mathbf{P}^{-1} , the inverse of the matrix **P**. [2 marks]

Answer _____

(ii) Hence find the matrix **X** such that
 $\mathbf{PX} = \mathbf{Q}$ [2 marks]

Show clearly each stage of your working.

Answer _____

6 (a) Express $3 \log 2x - 2 \log x$
as a single log. [3 marks]

Answer _____

(b) Solve the equation [5 marks]

$$4^{2x+1} = 6^{3x-2}$$

Answer _____

7 (a) Matrices **A**, **B** and **C** are defined by

$$\mathbf{A} = \begin{bmatrix} 4 & 5 \\ -1 & -1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix} \text{ and}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 5 \\ 0 & 3 \end{bmatrix}$$

Calculate

(i) $\mathbf{B} - 2\mathbf{C}$ [1 mark]

Answer _____

(ii) A^2 [2 marks]

Answer _____

(b) Find the values of a , b and c in the following matrix equation. [3 marks]

$$\begin{bmatrix} 2 & -1 \\ a & 1 \end{bmatrix} \begin{bmatrix} 4 & b \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ c & 6 \end{bmatrix}$$

Answer $a =$ _____

$b =$ _____

$c =$ _____

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(Questions continue overleaf)

8 Simplify **fully** the following algebraic expressions.

(a) $\frac{2x^2 + 6x}{x^2 - 1} \div \frac{4x + 12}{2x^2 + 3x - 5}$ [4 marks]

Answer _____

(b) $\left(\frac{x}{x+3} + \frac{2}{x^2-9}\right) \times \frac{x+3}{x-1}$ [5 marks]

Answer _____

9 At the point P, where $x = -1$, the tangent to the curve $y = 4 + \frac{a}{x^2}$ is parallel to the straight line $y = 3x + 1$

(i) Find the value of a . [4 marks]

Answer _____

(ii) Find the equation of the tangent to the curve at the point P. [3 marks]

Answer _____

(iii) Find the equation of the normal to the curve at the point P. [2 marks]

Answer _____

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(Questions continue overleaf)

10 A curve is defined by the equation

$$y = 2x^3 - 5x^2 - 3x$$

- (i)** Find the **coordinates** of the points where the curve meets the x -axis. [3 marks]

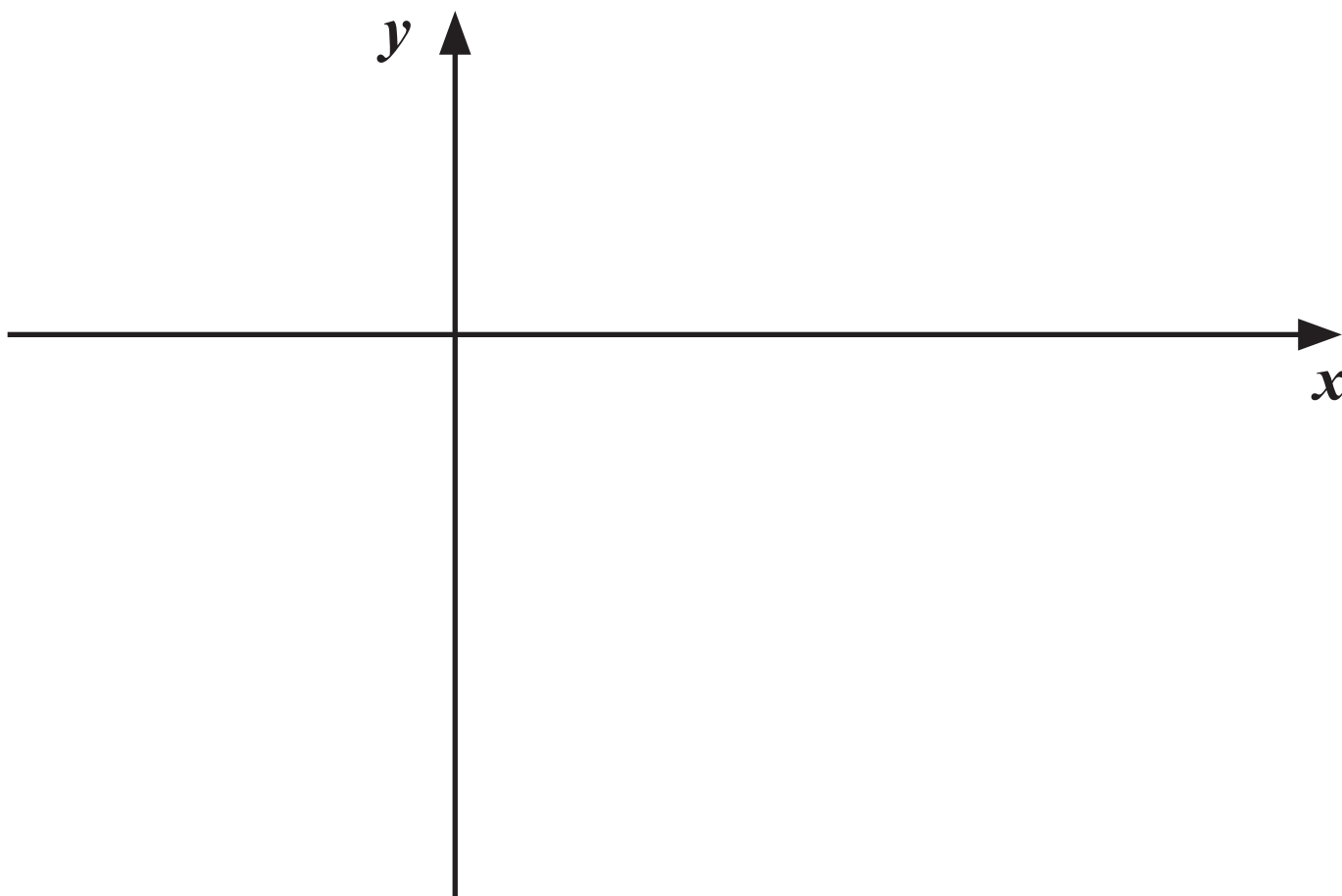
Answer _____

(ii) Find the coordinates of the turning points of the curve and, using calculus, identify each turning point as either a maximum or minimum point. [7 marks]

You **must** show working to justify your answer.

Answer _____

(iii) Sketch the curve on the axes below.
[2 marks]



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(Questions continue overleaf)

11 A car showroom sells the new car models Hurricane, Iguana and Jeronimo.

The profit for each Hurricane sold is x ,

the profit for each Iguana sold is y ,

the profit for each Jeronimo sold is z ,

where x , y and z are in thousands of pounds.

In February, 10 Hurricanes, 12 Iguanas and 16 Jeronimos were sold.

The total profit was £94,000

(i) Show that x , y and z satisfy the equation
[1 mark]

$$5x + 6y + 8z = 47$$

In March, 16 Hurricanes, 6 Iguanas and 4 Jeronimos were sold.

The total profit was £82,000

(ii) Show that x , y and z also satisfy the equation [1 mark]

$$8x + 3y + 2z = 41$$

In April, 18 Hurricanes, 3 Iguanas and 12 Jeronimos were sold.

The total profit was £96,000

(iii) Show that x , y and z also satisfy the equation [1 mark]

$$6x + y + 4z = 32$$

(iv) Solve the equations

$$5x + 6y + 8z = 47$$

$$8x + 3y + 2z = 41$$

$$6x + y + 4z = 32$$

to find the profit the showroom makes for the sale of each model of car.

[8 marks]

Show clearly each stage of your solution.

Answer Hurricane £ _____

Iguana £ _____

Jeronimo £ _____

12 The table below shows the power, P (watts), and the voltage, V (volts), in a particular circuit.

Power P (watts)	Voltage V (volts)		
11	1.5		
45	3		
125	5		
245	7		
405	9		

It is believed that a relationship of the form

$$P = kV^n$$

exists, where k and n are constants.

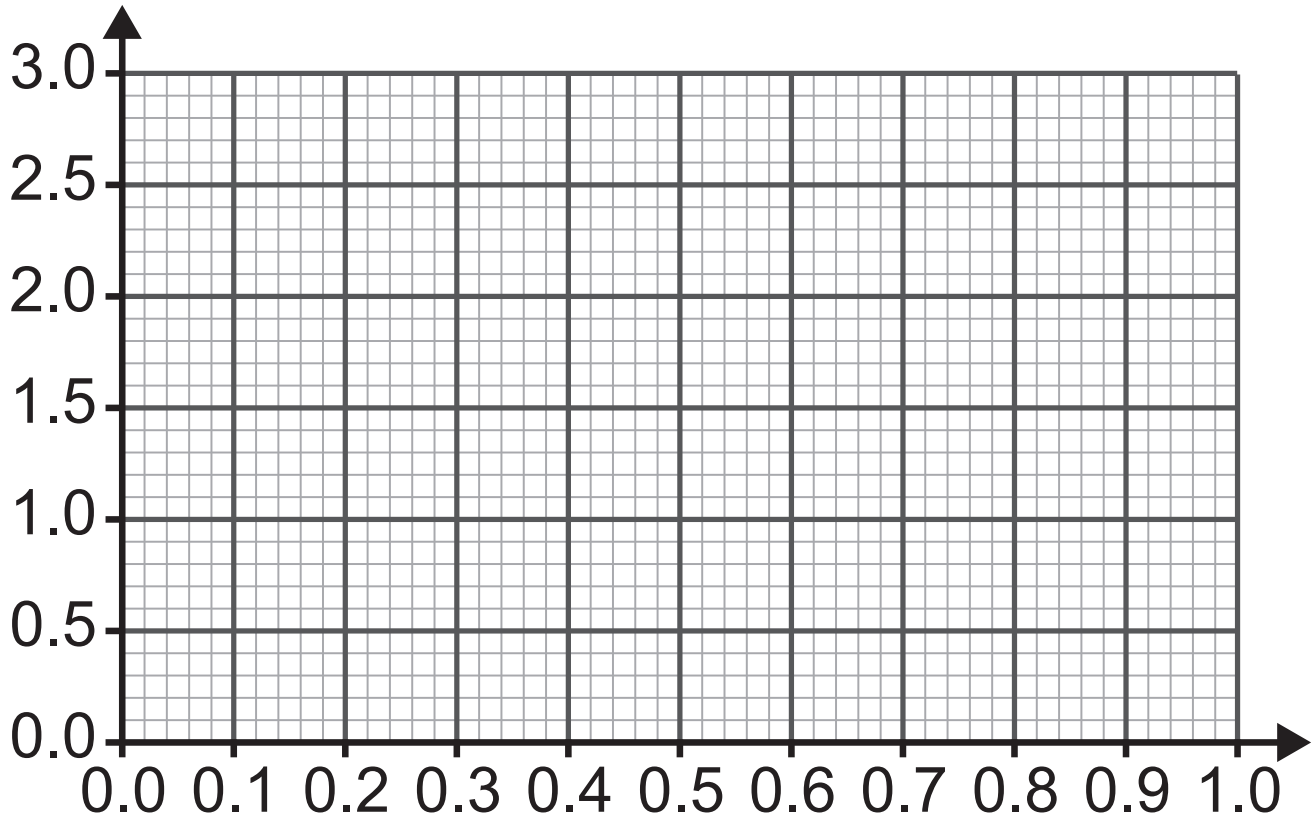
(i) Verify that a relationship of the form

$$P = kV^n$$

exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table opposite.

Label the axes clearly. [6 marks]



(ii) Hence find the values of k and n , correct to 2 decimal places. [4 marks]

Answer $k =$ _____ ,

$n =$ _____

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(Questions continue overleaf)

(iii) Use the formula $P = kV^n$, with your values for k and n , to calculate the minimum voltage required if the power must exceed 1000 watts.

Give your answer correct to the nearest volt and state any assumption that you make.

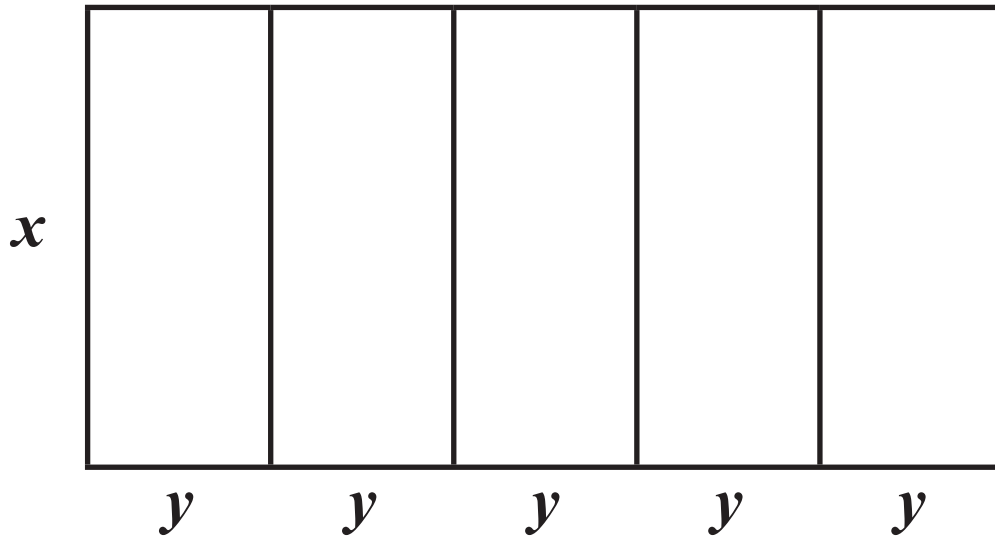
[2 marks for volts, 1 mark for assumption]

You **must** show your working.

Answer _____ volts

Assumption _____

13 A gardener fences 5 adjacent plots, each x m by y m, with shared fencing, as shown in the diagram below.



A total of 300 m of fencing is available.

(i) Show that $y = 30 - \frac{3}{5}x$ [1 mark]

(ii) Hence show that the total area of the 5 plots is [2 marks]

$$A = 150x - 3x^2$$

(iii) Find the maximum total area of the 5 plots, proving by calculus that it is a maximum. [5 marks]

Answer _____ m²

This is the end of the question paper

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	

Total Marks	
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Examiner Number

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